

A manifestation of a gluon saturation in e-A DIS

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This is a short presentation of our talks given at eRHIC Workshop at the BNL. We give here a status report of our attempts to understand how gluon saturation will manifest itself in deep inelastic scattering with nuclei. This summary reflects our current understanding and shows directions of our research rather than a final answer to the question. Nevertheless, we are able to share with our reader our tentative answer to the question: “Why do we need to measure DIS with nuclei and why these data will be complementary to the information obtained from proton DIS”.

I. INTRODUCTION: WHAT ARE THE SCALES IN PHOTON-NUCLEUS DIS?

The main goal of these notes is to examine if and how we can observe the phenomenon of gluon saturation in DIS with nucleus. We present here only a status report of our attempts to clarify this subject, which is far away from being complete. It, rather, indicates the directions of our searches. Much more work is needed to develop a reliable approach so as to finalize our recommendations concerning experiments the most sensitive to the gluon saturation.

We start with the general approach to photon-nucleus interaction, developed by Gribov [1] who suggested following time sequence of this process:

1. First, the γ^* fluctuates into a hadron (quark - antiquark) system well before the interaction with the target;
2. Then the converted quark-antiquark pair (or hadron system) interacts with the target.

Generally, these two stages result in the following formula for the cross section

$$\sigma_{tot}(\gamma^* + A) = \sum_n |\Psi_n|^2 \sigma_{tot}(n + A; x) , \quad (1)$$

where Ψ_n is the wave function of the system, produced in the first stage of the process.

A. Separation scale $r_\perp^{\text{sep}} \approx 1/M_0$

This scale is a typical distance which separates the pQCD approach from the non-perturbative one. Roughly speaking, for shorter distances than r_\perp^{sep} , the QCD running coupling constant can be considered as a small parameter while for longer distances $\alpha_S(r_\perp)$ is large and we cannot use the powerful methods of pQCD. Table 1 demonstrates how this scale works in our particular model to incorporate the long distance physics [2–4].

Table 1

Perturbative QCD	→	non-perturbative QCD
short distances	→	long distances
$r_{\perp} <$	r_{\perp}^{sep}	$< r_{\perp}$
DOF: colour dipoles [5]	•	DOF: constituent quarks [6]
Ψ_n : QED for virtual photon	•	Ψ_n : generalized VDM for hadronic system
$\sigma_{tot}(n, x) = \sigma(r_t^2, x)$	•	$\sigma_{tot}(n, x) = \sigma(qq \rightarrow qq; x)$
Glauber- Mueller Eikonal [7] for $\sigma(r_{\perp}^2, x)$	•	Regge phenomenology for $\sigma(q + q \rightarrow q + q; x)$

It is important to notice that the separation scale mostly relates to the produced hadronic ($q\bar{q}$) system and does not depend on the properties of the target (in particular, the atomic number). From Table 1 one can write for short distances ($r_{\perp} < r_{\perp}^{sep}$)

$$\sigma_{tot}(\gamma^* p) = \int d^2 r_{\perp} \int_0^1 dz |\Psi(Q^2; r_{\perp}, z)|^2 \sigma_{tot}(r_{\perp}^2, x). \quad (2)$$

B. Saturation scale $r_{\perp}^{sat} \approx 1/Q_s(x; A)$

At low x and at $r_{\perp} < r_{\perp}^{sep}$ we believe [8] that *The system of partons always passes the stage of hdQCD (at shorter distances) before it goes to the black box, which we call non-perturbative QCD, and which, in practice, we describe in old fashion Reggeon phenomenology.* At the hdQCD stage we have to observe a parton system with sufficiently small typical distances ($r_{\perp}^{sat} \approx 1/Q_s(x; A)$) at which the QCD coupling constant is still small ($\alpha_S(r_{\perp}^{sat}) \ll 1$), but the density of partons is so large that we cannot use the pQCD methods in our calculations. The picture of the parton distribution in the transverse plane is shown in Fig. 1.

The estimate of the value for the saturation scale is obtained [8] from the equation

$$\kappa = \frac{3\pi^2\alpha_S A}{2Q_s^2(x)} \times \frac{xG(x, Q_s^2(x))}{\pi R_A^2} = 1, \quad (3)$$

where A and R_A are the atomic number and radius of the nucleus. Eq. (3) has a simple physical meaning giving the probability of the interaction between two partons in the parton cascade. Namely, such an interaction will stop the increase of the parton density due to parton emission, which is included in the DGLAP evolution equations [9].

It is important to notice that the saturation scale strongly depends on A ($Q_s(x; A) \propto A^{\frac{1}{6}}$ [10] $\div A^{\frac{1}{3}}$ [11]).

C. The theory status

In eA deep inelastic scattering we want to find the high density parton system which is a non-perturbative system but which can be treated theoretically. It should be stressed that the theory of hdQCD is in a very good shape now. Two approaches have been developed for hdQCD: the first one [12] is based on pQCD (see GLR and Mueller and Qiu in Ref. [8]) and on the dipole degrees of freedom [5], while the second [13] uses the effective Lagrangian, suggested by McLerran and Venugopalan [8]. As a result of this intensive work we know now the nonlinear equation which governs the QCD evolution in the hdQCD region [14]. We have not developed simple methods to estimate an effect of hdQCD on the experimental observables and have to use a model approximation, but we want to emphasize that this is a temporary stage of our theory which will be overcome soon.

II. HERA: RESULTS AND PUZZLES.

We start answering the question: “why do we need a nuclear target to find the hdQCD phase” with a summary of what we have learned from HERA.

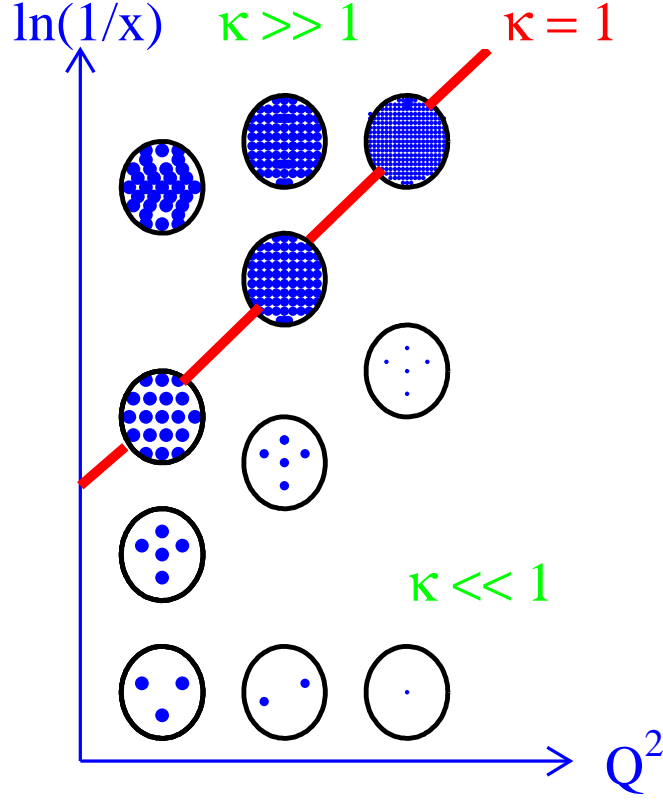


FIG. 1. The parton distribution in the transverse plane. The curve shows the saturation scale $Q_s(x; A)$

- **F_2** - the most striking and significant result from HERA is the increase of F_2 at low x [15]. The interpretation of the F_2 data in terms of the DGLAP evolution equations leads to sufficient large value and a steep behaviour of the gluon structure function at low x . $xG(x, Q^2)$ turns out to be so large that κ , our new order parameter, exceeds unity in a significant part of the accessible phase space (see Fig. 2).

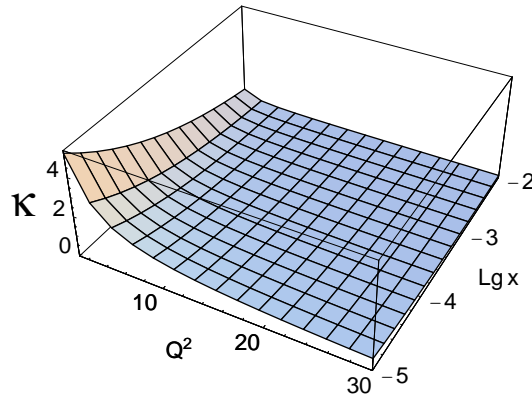


FIG. 2.

- **Diffractive production** - three important results have been observed at HERA: (i) the diffractive production gives a substantial part of the total cross section, about $10 \div 15$ % at $Q^2 \approx 10 \text{ GeV}^2$; (ii) the energy behaviour of the diffractive cross section has an intercept larger than the intercept of the soft Pomeron, namely, $\sigma_{diff} \propto (1/x)^{2\Delta_P}$ with $\Delta_P > \Delta_{softP} \approx 0.1$ [16], and (iii) the ratio $\sigma_{diff}/\sigma_{tot}$ is a constant versus energy in the HERA kinematic region. From Fig. 1, one can see that a hadron looks as a diffractive grid with typical size of the order of r^{sat} . Therefore, we expect that diffractive processes originate from a rather small distance. This fact leads to a natural explanation of the energy behaviour of the diffractive cross section.
- **Matching between soft and hard processes.** The experimental data on γ^*p cross section at small Q^2 allows to test different models for the matching of the soft and hard interactions.
- The dedicated beautiful **measurement of the F_2 slope** ($dF_2/\ln Q^2$) gives us a hope to find the saturation scale by observing the movement of the maxima in Q^2 - behaviour at fixed x . The experimental data show a considerable deviation from the DGLAP analysis at $Q^2 \leq 1 \div 3 \text{ GeV}^2$. However, the current data can be described in two different ways, either due to a gluon saturation or due to a probable matching between soft and hard at rather large momenta (about 1 - 2 GeV) [4]. It should be noticed, however, that the J/Ψ production can be easily described taking into account shadowing corrections confirming a gluon saturation hypothesis [4].

We listed above the most important HERA observations which indicate a possible saturation effect. To illustrate this fact we will demonstrate that a simple parameterization of Golec-Biernat and Wusthoff [17], which includes the saturation, works well. They found an elegant phenomenological model for $\sigma(r_\perp^2, x)$ in Eq. (2) which is able to describe all experimental data using only three parameters [17]. In this model

$$\sigma_{dipole}(r_\perp, x) = \sigma_0 \left(1 - \exp \left(-\frac{r_\perp^2 Q_0^2}{(x/x_0)^\lambda} \right) \right), \quad (4)$$

with $\sigma_0 = 23.03 \text{ mb}$, $Q_0^2 = 1 \text{ GeV}^2$, $x_0 = 0.0003$ and $\lambda = 0.288$. Figs. 3 and 4 show the quality of this description.

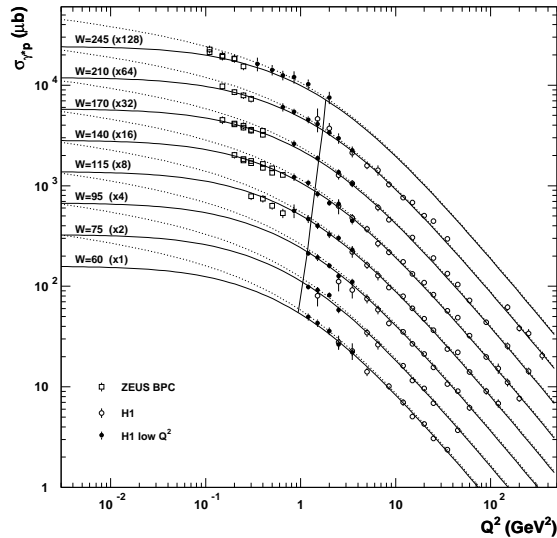


FIG. 3.

Note that even though the above description is impressive, it cannot fix the value of the saturation scale from the data which is too constrained by the kinematics (see Ref. [4] for details).

Therefore, we can conclude that the saturation hypothesis is compatible with all experimental data. However, the puzzling situation is that the same data can be described from a different point of view without a saturation scale in the standard DGLAP evolution equation for $Q^2 > 1\text{GeV}^2$ and the soft phenomenology for $Q^2 < 1\text{GeV}^2$. We do not claim that it is a reasonable or smooth parameterization of the data but Donachie-Landshoff mixture of soft and hard Pomeron shows that we can produce such a model.

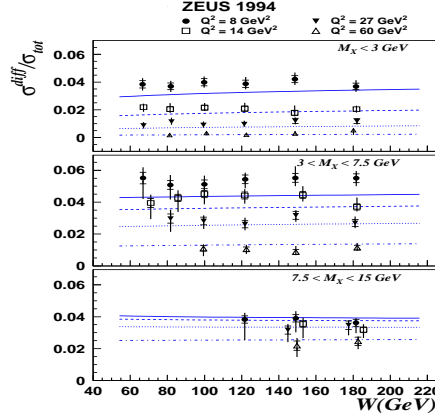


FIG. 4.

Thus in order to fix the saturation scale and to discriminate between competing models, we need either to reach a much smaller values of x (higher energy) or to use a new target. Realistically, we can conclude that

We need DIS with nuclei to check whether the indications on saturation effect at HERA are really true.

III. SCALE OF THE SATURATION EFFECT FOR DIS WITH NUCLEI

A. Asymptotic predictions.

Let us start with listing the asymptotic predictions of our approach [2–4] which is based on the Glauber-Mueller formula for $\sigma(r_\perp^2, x)$ in Eq. (2) [7].

- At fixed r_\perp and at $x \rightarrow 0$

$$\sigma_{tot}^{dipole} \longrightarrow 2\pi \left(R_A + \frac{h}{2} \ln(Q_s^2(x; A)/Q^2) \right)$$

where R_A is the nucleus radius and h is the surface thickness in the Wood-Saxon nucleon density ;

- In the same limit $\frac{dF_2^A}{d\ln Q^2} \longrightarrow F_2^A \left(1 - \frac{h}{R_A} \right) \propto Q^2 R_A^2$;
- The ratio of the diffraction to the total cross sections should depend on energy only weakly [18];

$$\frac{\sigma_{tot}^{diffraction}(\gamma^* A)}{\sigma_{tot}(\gamma^* A)} \approx \text{Const}(W) \longrightarrow (\text{slowly}) \frac{1}{2};$$

- The energy behaviour of $\sigma^{diffraction}(\gamma^* A)$ is determined by short distances $r_\perp \approx 1/Q_s(x; A)$;
- The high density effects should be stronger in the diffractive channels.

B. $xG_A(x, Q^2)$

In Fig.5 we present our calculation of the gluon structure function for different nuclei. Fig.5-d gives a glimpse at what we are taking into account in our approach. Figs. 5-a - 5-c show the comparison of our calculations, based on the Glauber-Mueller formula, with the solution of the full equation for hdQCD [14] ¹

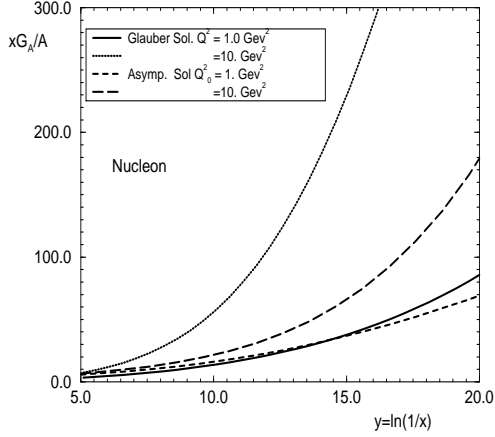


Fig.5-a

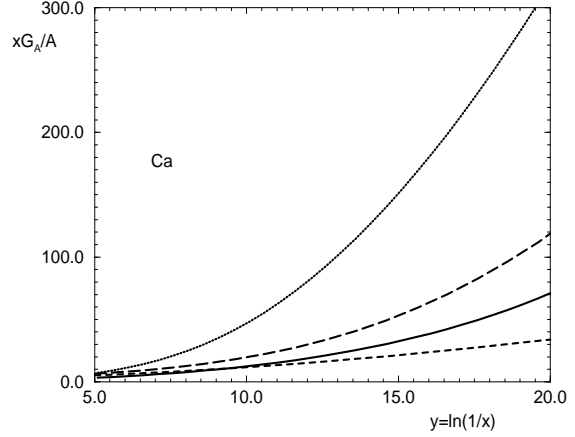


Fig.5-b

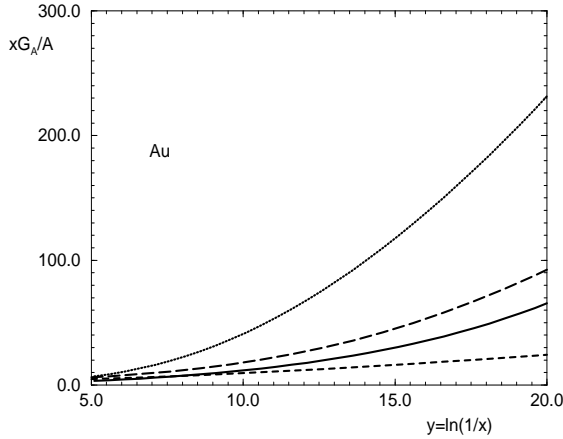


Fig. 5-c

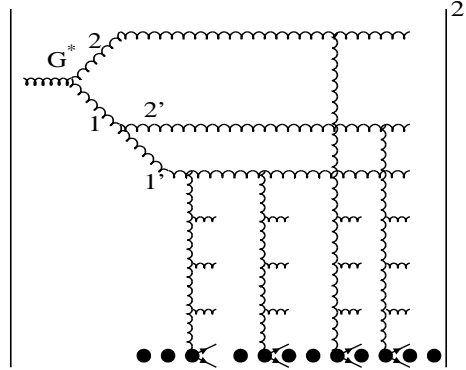


Fig.5-d

FIG. 5.

We can derive two conclusions from Fig.5: (i) the saturation effect is much stronger for a nuclear target than for a nucleon, and (ii) our model underestimates the value of the effect for $Q^2 \approx 1 \text{ GeV}^2$. Unfortunately, we have not finished our estimates for F_2 for DIS with nuclei.

¹Actually, the equation, suggested in Ref. [3] was solved and plotted in Fig.5 as the asymptotic solution, but this equation in the double log approximation coincides with the correct one [14].

$$\text{C. } R = \frac{\sigma_{\text{tot}}^{\text{diffraction}}}{\sigma_{\text{tot}}}$$

This ratio shows us how we are close, or how we are far away, from the asymptotic regime since at very high energy it should be equal to $\frac{1}{2}$. In Fig.6 we plotted our calculations for this ratio [19]. One can see that the ratio is larger than for the proton target (see Fig. 4) but it is still smaller than the limiting value of $\frac{1}{2}$. This is a very encouraging fact for experiment since we do not want to measure a black disc limit which is not sensitive to the theoretical approach. In other words, any model or any theoretical approach will give the unitarity limit which we call “ black disc limit”. All our theoretical QCD prediction are related to the form of the transition from pQCD to the “black disc limit”.

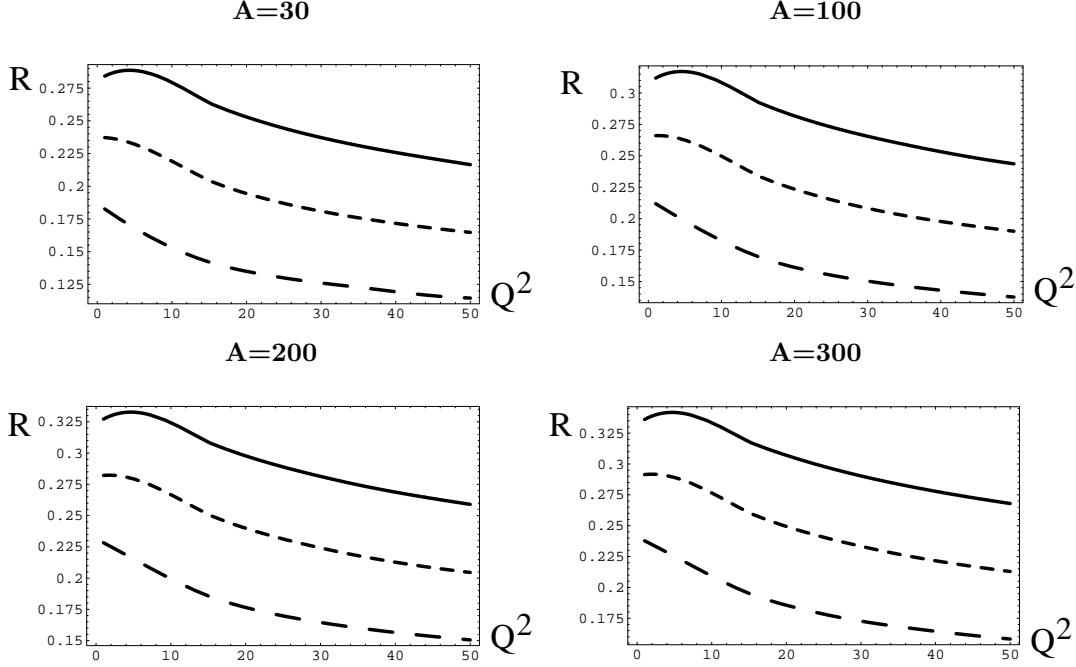


FIG. 6.

$$\text{D. } F_2 \text{ slope } \left(\frac{\partial F_2^A(x, Q^2)}{\partial \ln Q^2} \right)$$

As we have mentioned, gluon saturation leads to a maximum in the Q^2 dependence of F_2 slope at $Q^2 = Q_s^2(x; A)$ for a fixed value of x . Such a maximum has not been seen in the HERA data and has not been anticipated in our estimates of the slope. However, the numerical value of the gluon saturation is rather for a nucleus target. Figs. 7 and 8 display the possible experimental effect. In these figures the value of the damping factor (D^A) for the F_2 slope is plotted. D^A is defined in the following way:

$$\frac{dF_2^A(x, Q^2)}{d \ln Q^2} = D^A(x, Q^2) A \frac{dF_2^{N;DGLAP}(x, Q^2)}{d \ln Q^2}, \quad (5)$$

where $F_2^{N;DGLAP}$ is the F_2 structure function for a nucleon in the DGLAP approximation. It turns out that the value of the effect is sizable for $Q^2 < 10 \text{ GeV}^2$ and strongly depends on A .

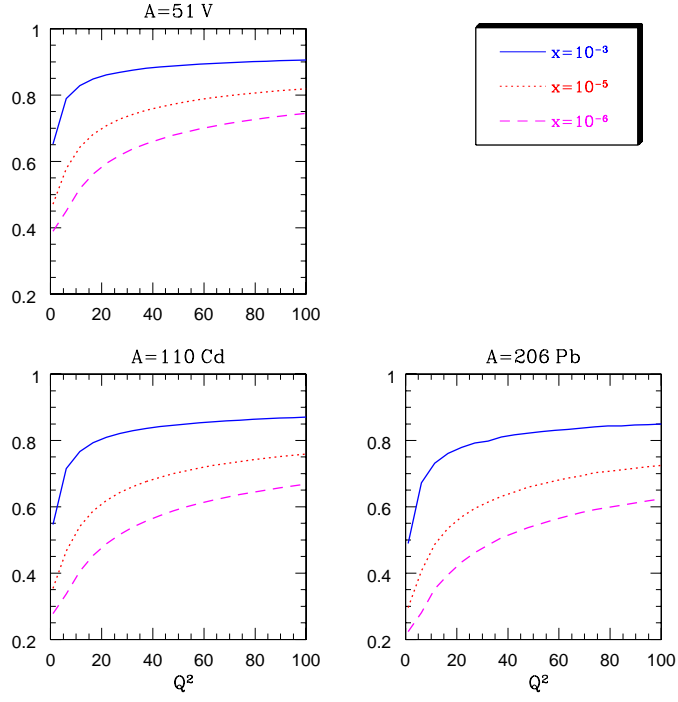


FIG. 7.

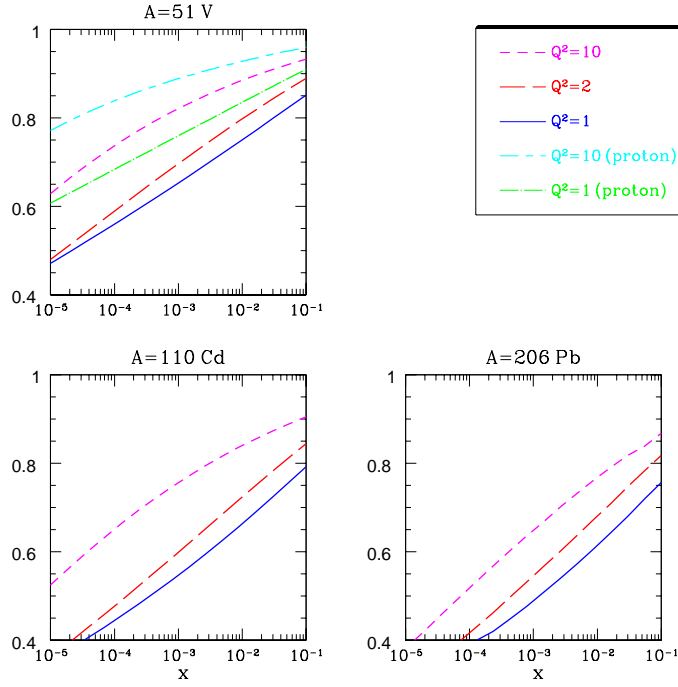


FIG. 8.

IV. SEARCHING NEW OBSERVABLES.

A. Maxima in $\frac{F_L}{F_T}$ and in $\frac{F_L^D}{F_T^D}$.

The main idea of this calculation [20] is to show that these ratios have maxima at $Q^2 = Q_{max}^2(x; A)$, being plotted at fixed x . We want to claim that $Q_{max}^2(x; A) \approx Q_s^2(x; A)$. Figs. 9 and 10 show that such a suggestion can be right.

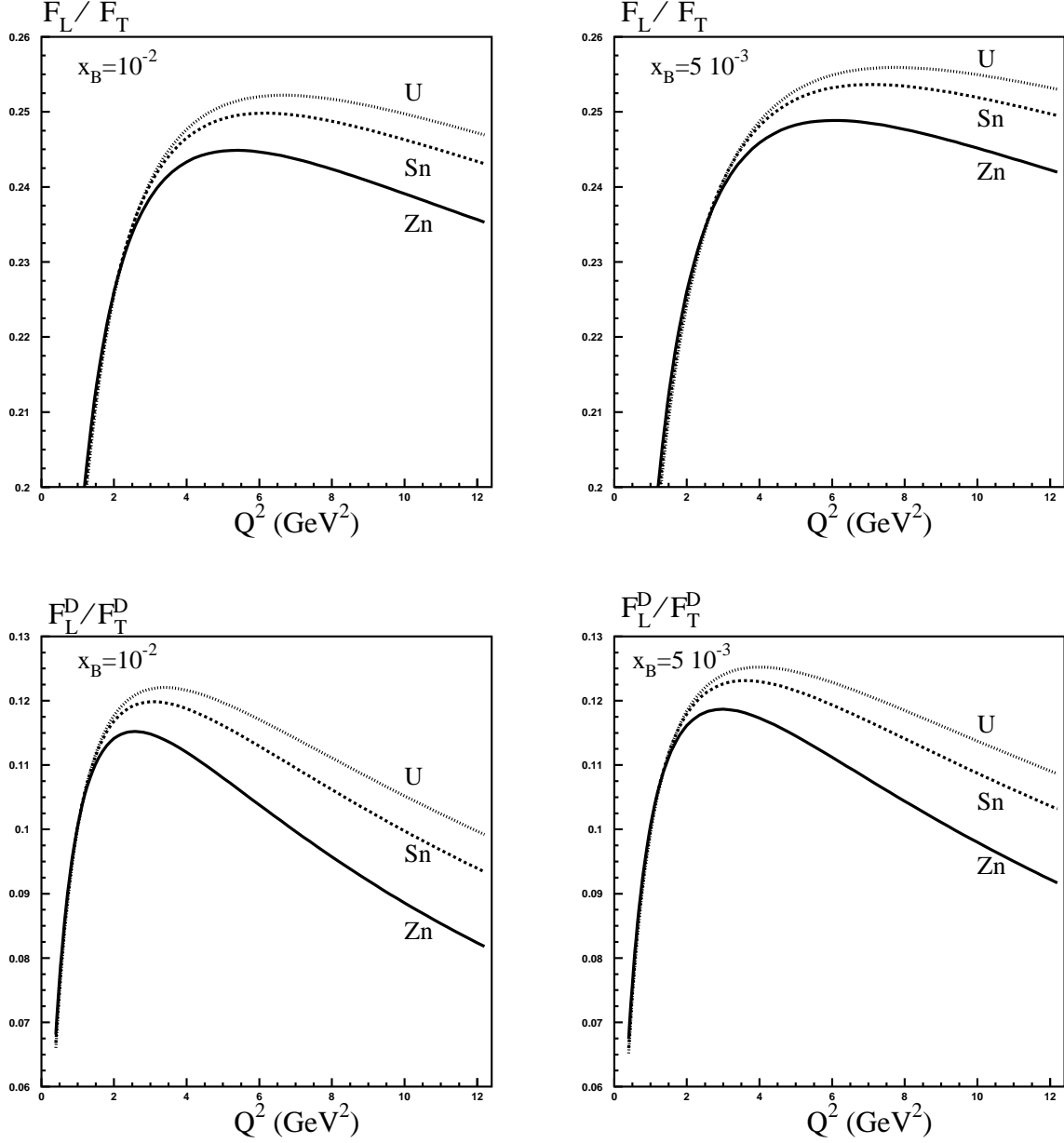


FIG. 9.

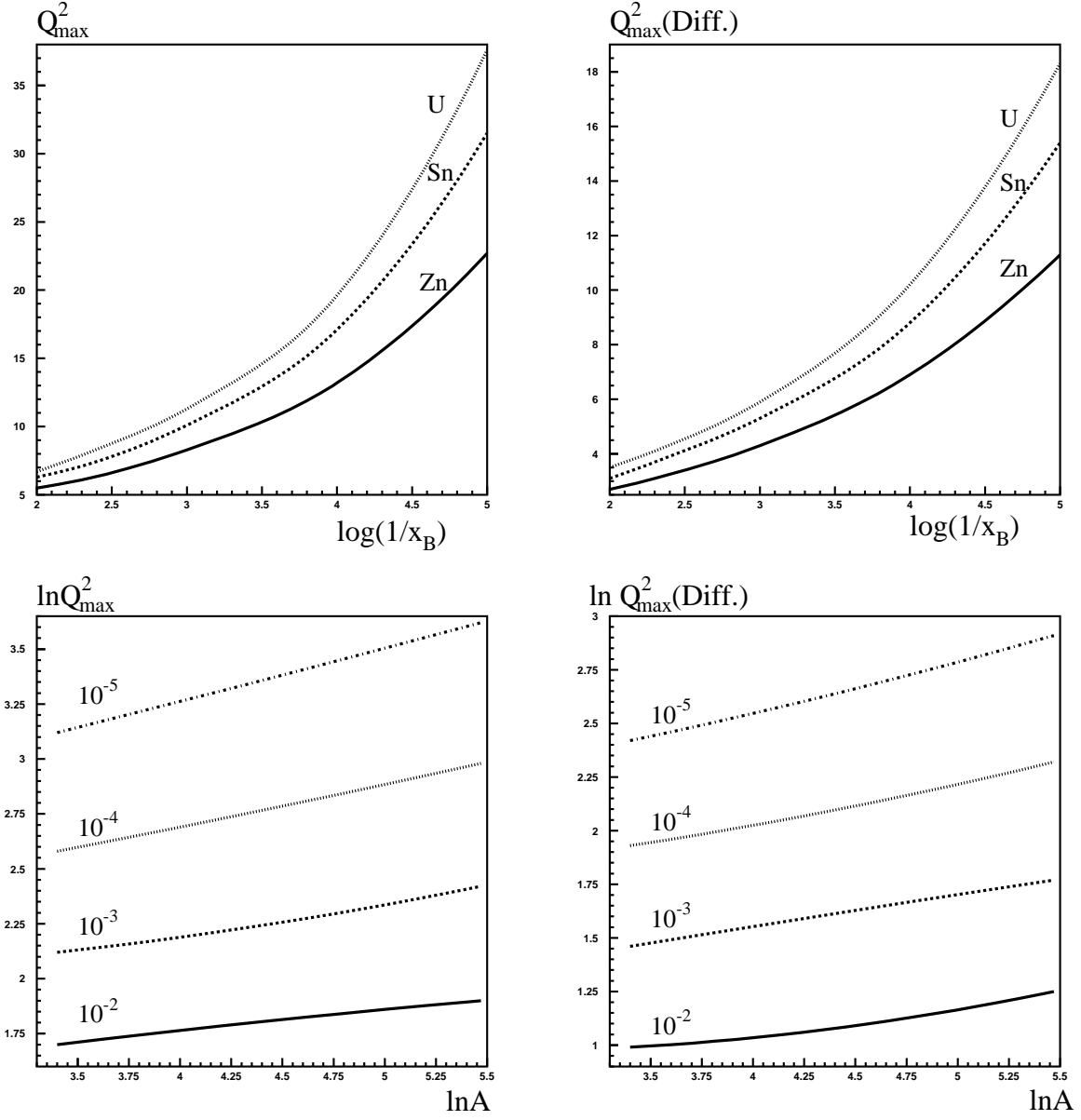


FIG. 10.

B. Higher twists in $F_{L,T}$ and in $F_{L,T}^D$.

It is well known that any structure function ($F_2(x, Q^2)$ for example) can be written

$$F_2(x, Q^2) = F_2^{LT}(x, \ln Q^2) + \frac{M^2}{Q^2} F_2^{HT}(x, \ln Q^2) + \dots + \left(\frac{M^2}{Q^2} \right)^n F_2^{nT}(x, \ln Q^2) + \dots \quad (6)$$

Terms, which are small in terms of Q^2 power, called higher twist contributions. The $\ln Q^2$ dependence of the leading and higher twist structure functions (F_2^{LT} and F_2^{HT} in Eq. (6)) is governed by the evolution equations. The DGLAP evolution equations [9] give the $\ln Q^2$ dependence of the leading twist structure function (F_2^{LT}) only. Unfortunately, we know only a little about the higher twist contributions.

1. We know the evolution equations for all higher twist structure functions [21];
2. We know the behaviour of the higher twist structure functions at low x [22]. For example,

$$F_2^{HT}(x, \ln Q^2)|_{x \ll 1} \longrightarrow F_2^{LT}(x, \ln Q^2) \cdot x G^{LT}(x, \ln Q^2) ;$$

3. We know, that higher twist contributions are needed to describe the experimental data [23].

However, it is difficult to estimate the value of the higher twist contributions. Following Ref. [24], we estimate the value of different twist contribution for e A scattering.

Fig.11 shows that the higher twist contributions for nucleus target become smaller than the leading twist one only at $Q^2 > 5 GeV^2$ even at $x = 10^{-2}$. It gives us a hope to treat them theoretically.

V. CONCLUSIONS.

1. We have a solid theoretical approach for eA DIS, but we need more experience in numerical solution of the non-linear equation specifically for eRHIC kinematic region;
2. We know pretty well the scale of SC for eA interaction, but we need more systematic study of DGLAP evolution for nuclear structure functions and a special investigation whether the initial parton distributions could be calculated for nuclear target from the initial parton distributions for proton ;
3. Our estimates show that we will be able to see the saturation scale in eA DIS at eRHIC being still far away from trivial blackening of high energy interaction with nuclei, but we need to check how close our model, which we use in practise, to theoretical estimates;
4. The F_2^A -slope is a very sensitive observable for the saturation scale, but , unfortunaly, we cannot expect a qualitatively different behaviour for the saturation models in comparison with others;
5. Maxima in ratios of F_L/F_T and F_L^D/F_T^D give promising tool to extract the value of saturation scale $Q_s(x; A)$, but we need more study on this subject and, in particular, how the initial parton distribution for DGLAP evolution could affect our predictions;
6. eA DIS is very instructive for separation of leading and higher twist contributions, since the fact that typical momentum at which these two contributions become of the same order is growing with A .

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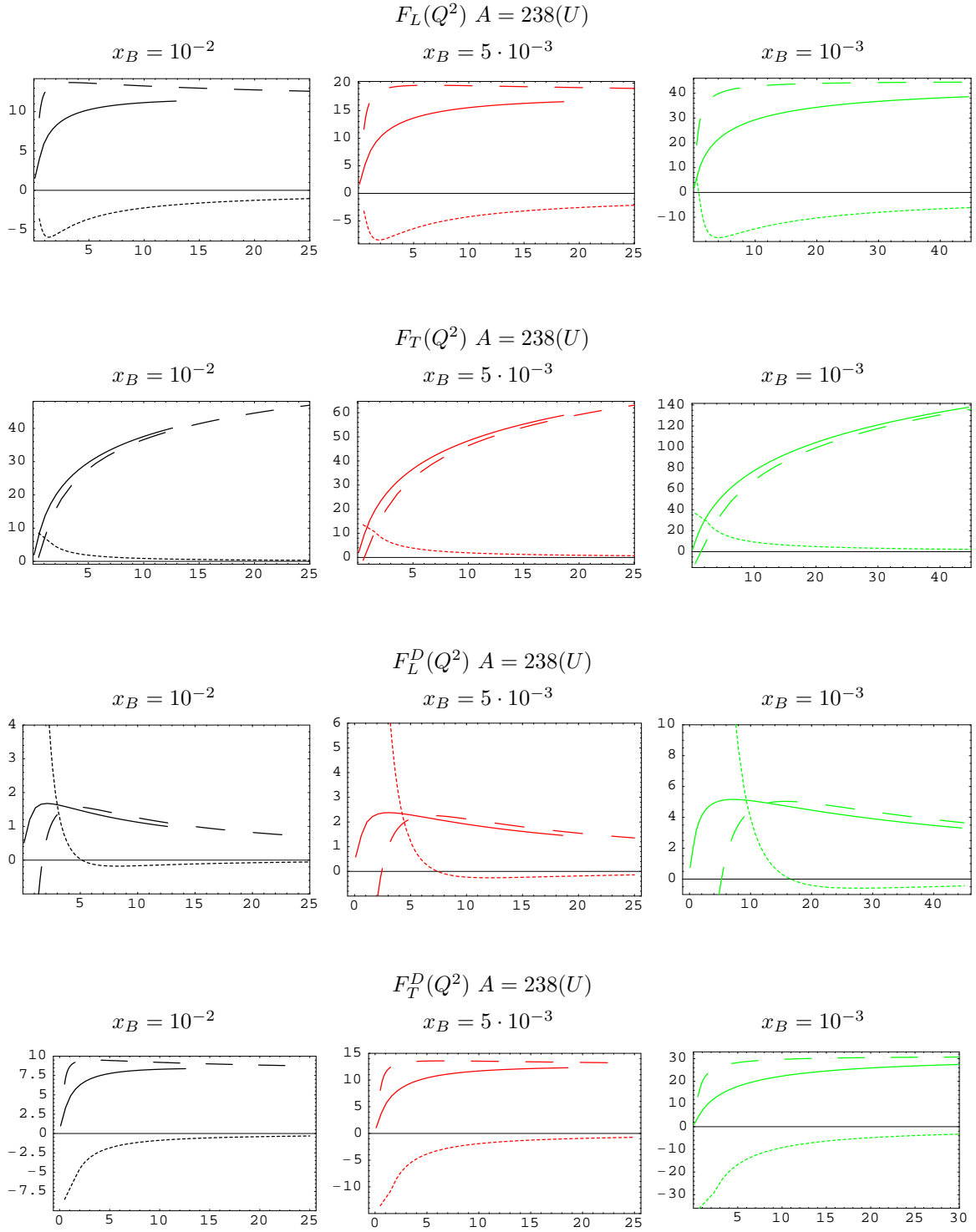


FIG. 11.

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